

Physics 798C
Superconductivity
Fall 2025
Homework 3
Due Thursday 9 October, 2025

1. Equivalence of Creation/Annihilation wavefunctions and Slater Determinants

We want to show that the creation/annihilation operator format for a wavefunction is entirely equivalent to the (more laborious) Slater determinant version of the wave function. Consider the two-particle Cooper pairing wavefunction in the creation/annihilation operator format:

$$|\psi_0\rangle = \sum_{k > k_F} g_{\vec{k}} c_{\vec{k}\uparrow}^+ c_{-\vec{k}\downarrow}^+ |F\rangle,$$

where $|F\rangle$ represents the filled Fermi sea. Show that this is equivalent to the form of the Cooper pair wavefunction that we derived in class, Tinkham Eq. (3.1), by summing the two 2x2 Slater determinants with the equal coefficients $g_{\vec{k}}$ and $g_{-\vec{k}}$, which are equal by time-reversal invariance.

2. BCS Variational Calculation

Starting from the BCS pairing Hamiltonian in terms of the $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$, work through the variational calculation and derive the final results for $u_{\mathbf{k}}$, $v_{\mathbf{k}}$, and the zero temperature gap Δ . This was outlined in Lecture 9. In other words, start from this equation:

$$\langle \Psi_G | \mathcal{H} - \mu N_{op} | \Psi_G \rangle = 2 \sum_k \xi_k |v_k|^2 + \sum_{kl} V_{kl} u_k v_k^* u_l^* v_l$$

... and arrive at these equations:

$$u_k^2 = \frac{1}{2} \left[1 + \frac{\epsilon_k - \mu}{\sqrt{\Delta^2 + (\epsilon_k - \mu)^2}} \right] \text{ and } v_k^2 = \frac{1}{2} \left[1 - \frac{\epsilon_k - \mu}{\sqrt{\Delta^2 + (\epsilon_k - \mu)^2}} \right]$$